TOPIC: Electrostatics and Magnetostatics (ENEL475)

Q.1 We are interested in finding the potential $V(\overrightarrow{\mathbf{r}})$ at a point on the z-axis $P = (0, 0, z)$ for a uniform charge density ρ_S distributed on a disk of radius $r = a$ lying in the xy-plane and centred around the origin, assuming the reference is chosen as infinity.

(a) Draw a clear sketch of this problem to analyze the geometry. Your sketch should clearly indicate the field point P, the field position vector $\overrightarrow{\mathbf{r}}$, and the source position vector $\overrightarrow{\mathbf{r}}'$.

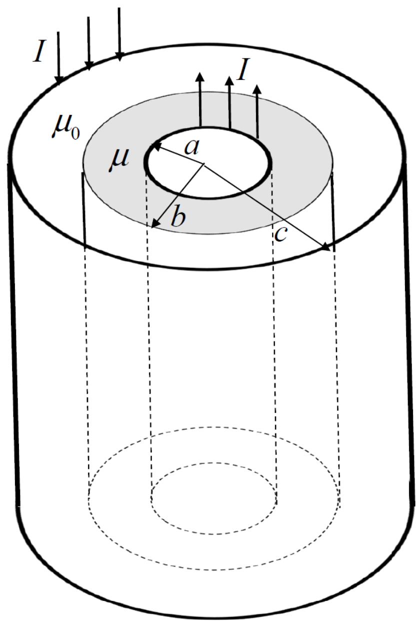
(b) Solve for the potential $V(\overrightarrow{\mathbf{r}})$ at a point on the z-axis $P = (0, 0, z)$
Note: You will probably require one of the following integrals in order to solve this problem

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right) \qquad \int \frac{x}{\sqrt{x^2+a^2}}dx = \sqrt{x^2+a^2}$$

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) \qquad \int \frac{x}{x^2+a^2}dx = \frac{1}{2}\ln\left(x^2+a^2\right)$$

Q.2

An infinitely long coaxial cable has a *hollow* inner conductor of radius a, which has a cladding of radius b which is a magnetic material of permeability μ, as depicted in the figure below. The inner conductor carries the *total* supply current I, and the outer conductor carries the *total* return current I; both are distributed uniformly around their respective cylindrical surface in the directions indicated in the figure.



(a) Choose an appropriate coordinate system and, *using Ampere's law*, show that the magnetic field everywhere in the region $a < \rho < b$ is given by

$$\vec{\boldsymbol{H}} = \frac{I}{2\pi\rho}\hat{\mathbf{a}}_\phi \, \frac{\text{A}}{\text{m}}$$

(b) Using the given field in part (a), calculate the stored magnetic energy in the cladding region for a length d of the cable and for a current I.

(c) If the relative permeability of the cladding is $\mu_r = 20$, what is the inductance of a length d of this cable?

Q.3 Starting from Gauss' law in integral form, produce a derivation of the concept of Divergence by showing that

$$\nabla \cdot \vec{\boldsymbol{D}} = \lim_{\Delta v \to 0} \frac{\oint_S \vec{\boldsymbol{D}} \cdot d\vec{\boldsymbol{S}}}{\Delta v}$$

Q.4 Starting from a simple microscopic/atomic basis, explain how each of the following macroscopic properties is manifest:

(a) Conductivity σ

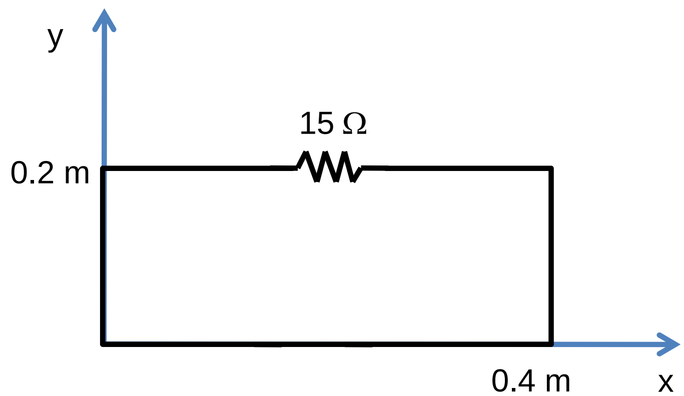
(b) Permittivity ε

(c) Permeability μ

TOPIC: Electromagnetic waves and applications (ENEL476)

Q.1 Consider the loop containing a resistor as shown below. The loop is placed in a magnetic flux density described by:

$$\mathbf{B}=-20\cos(100\pi t-\pi/3)\mathbf{a}_z \text{ mWb/m}^2$$



a) Find the EMF (V_{emf})
b) Calculate the induced current in the loop. Indicate the direction of current flow during the first quarter period on the figure above.

Q.2 A ground penetrating radar system is modeled as a uniform plane wave in free space impinging on the ground at normal incidence. The incident electric field (in free space, so properties are ε_o, μ_o, $\sigma=0$) is given by:

$$\mathbf{E}^i(x,t)=10\cos(10^9 t-3.3x)\mathbf{a}_y \text{ V/m}$$

a) Find the wavelength.

The ground has properties of $\varepsilon_r=4$, $\mu_r=1$, and $\sigma=0.1$ S/m.

b) Calculate the reflection (Γ) and transmission (T) coefficients.
c) Find an expression for the reflected electric field ($\mathbf{E}^r(x,t)$).
d) Find an expression for the transmitted electric ($\mathbf{E}^t(x,t)$) and magnetic fields ($\mathbf{H}^t(x,t)$).

Q.3 A distortionless transmission line has R=5 Ω/m, L=20 μH/m, C=30 nF/m and is operated at 10 MHz. Calculate the following quantities:

a) G

b) the impedance of the line, Z_o

c) the attenuation of the line, α

d) the phase constant of the line, β

e) the wavelength on the line, λ

A load of Z_L=30-j40 Ω is attached to 6 cm of the transmission line.

f) Using the appropriate equation, find Z_{in} for the section of transmission line terminated by the load.

Q.4 A load of impedance Z_L=30-j60 Ω is attached to a transmission line with 75 Ω characteristic impedance (Z_o=75 Ω). The frequency of operation is 5 GHz and the wavelength on the line is 6 cm. Use the Smith Chart to solve the following questions.

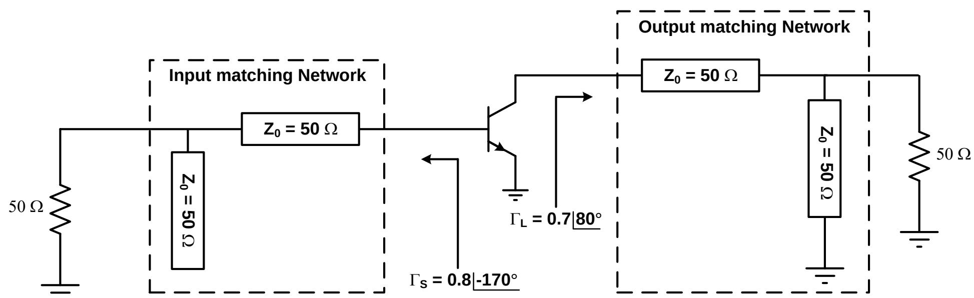
a) Find the reflection coefficient at the load (Γ).
b) Find the standing wave ratio (s). Verify with the appropriate equation
c) Find the input impedance Z_{in} for a line of length of 5.0 cm attached to the load.
d) Find the admittance of the load (Y_L).
e) Find the distance from the load to the first voltage minimum.
f) Find the shortest distance to a purely resistive load.

Q.5 A load of impedance Z_L=70+j25 Ω is attached to a transmission line with 100 Ω characteristic impedance (Z_o=100 Ω). The frequency of operation is 900 MHz and the wavelength on the line is 67 cm. To match the load to the line, design a series stub tuner with an open termination on the stub, and a shunt stub tuner with a short termination on the stub.

TOPIC: RF/Microwave Active Circuits (ENEL574)

Q.1 Determine the S parameters of two port network consisting of a series resistance R terminated at its input and output ports by the characteristic impedance Zo.

Q.2



a) Design the input matching network by giving the lengths of 50 Ohms transmission lines as function of the wavelength , λ as shown in the above figure to produce the source reflection coefficient Γ_S = 0.8 @ -170° at 1 GHz.

b) Design the output matching network by giving the lengths of 50 Ohms transmission lines as function of the wavelength , λ as shown in the above figure to produce the load reflection coefficient Γ_L = 0.7 @80° at 1 GHz.

Q.3 An amplifier is driven by modulated signal having a 20 MHz bandwidth is constituted by the cascade of three amplifiers A1, A2 and A3 and having the following characteristics:

Amplifier A1:	G_1=23 dB,	$IP3\_{}_1$=23 dBm,	NF_1=2 dB.
Amplifier A2:	G_2=17 dB,	IP3_2=42 dBm,	NF_2=3 dB.
Amplifier A3:	G_3=13 dB,	IP3_3=50 dBm,	NF_3=5 dB.

a) The parameters IP3_N are specified at the output of each amplifier and the reference noise temperature T_o is 290°K.
b) Determine the equivalent noise figure NF_e of the power amplifier as well as its equivalent noise temperature T_e.
c) Calculate the third order interception point IP_3 at the output of the power amplifier. Deduce the P_{1dB} of the power amplifier.

d) Determine the carrier to third order intermodulation products ratio (C/IMD_3) at the output the power amplifier when it is driven with a two-tone signal spaced by 10 MHz and having a total input power of -8 dBm

Q.4 The scattering and noise parameters of a GaAs FET transistor at 2 GHz are:

$S_{11} = 0.9\angle -60°$, $S_{21} = 3.1\angle 140°$, $S_{12} = 0.02\angle 62°$ and $S_{22} = 0.8\angle -27°$

$F_{min} = 1.5$ dB, $\Gamma_{opt} = 0.7\angle 55°$, $r_n = 0.95$

a) Study the stability of the device and draw the input and output stability circles in the Smith Chart,
b) Can the device be considered unilateral?
c) Draw the operating power gain circle for $G_P = 20$ dB.
d) Determine the source and load reflection coefficients required to design an amplifier to have an operating power gain of 20 dB. Explain your choices of Γ_S and Γ_L.

TOPIC: RF/Microwave Passive Circuits (ENEL575)

Q.1 An amplifier is attached to an antenna through a 9 cm long section of a coaxial cable. Assume the coax line has no loss, and its parameters are C=96 pF/m and L=240 nH/m. The antenna has a radiation impedance of 50 Ω (you can assume the antenna is your generator, and its radiation impedance the generator internal impedance), however the amplifier is not matched to the system. Measurements showed that the amp has an input impedance of (25+j20) Ω. The system operates at 10 GHz.

You have been asked to improve the system performance. Here are your tasks:

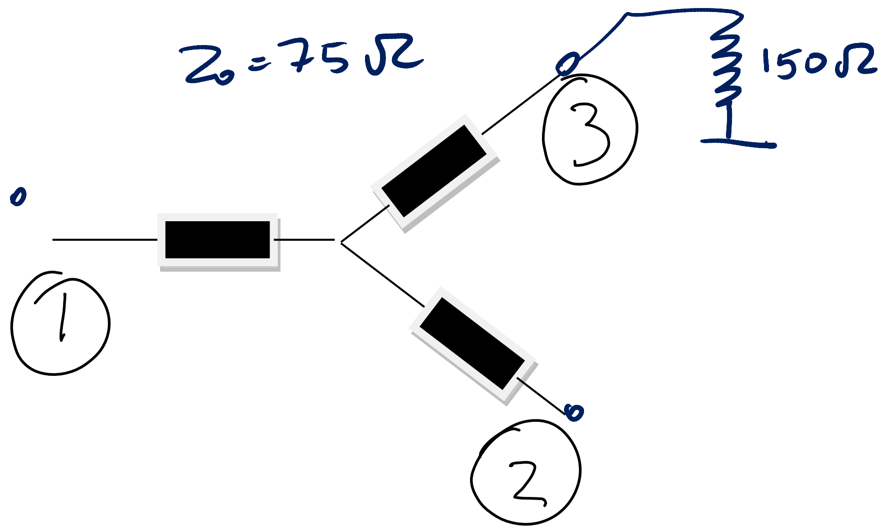
1. Assume that the antenna generates voltage Vg. Compute the voltage and the current at the input terminal of the transmission line (so from the antenna side, remember, your antenna acts as a generator here), find out the amplitudes of forward and backward travelling waves.
2. Compute available power. What percentage of available power at the generator is delivered to the amp?
3. Now match the system using lumped elements. What percentage of available power is delivered to the amp?

Remember to calculate the actual values of necessary inductances or capacitances.

Q.2 Consider a rectangular waveguide with the following dimensions: a=22.0mm, b=10.0mm. The waveguide walls are made of a perfect conductor, waveguide is filled with air (lossless). Frequency is 10 GHz

a. How much the TE_{20} mode will be attenuated at that frequency over a distance of 1 cm (expressed in dB)
b. The waveguide is loaded with an impedance of (154.6+j154.6)Ω. Match the system using double stub serial tuner. The tuners are short-circuited, and separated by 5/4λ (lambda). You are allowed to add 1 or 2 sections of λ/8 section of a waveguide between the tuner and the impedance (only if you find it necessary). Draw the structure, pay attention to the feasibility of your structure (that is, could you actually build it? Show all dimensions (in mm). In particular, show how the serial connections of the waveguide and the tuners look like.

Q.3 Design a three-port resistive divider for an equal power split and 75 ohm system impedance (lets' call this system A). Derive S^A matrix of this system. Now terminate port 3 of the 3-port system A with a 150 ohm resistor – effectively turning the initial 3 port structure into a 2 port structure with matrix S^B. Compute S^B_{21} in this new system.



Field of Study Examination, Feb 24 2017

Subject area: Radio Frequency Systems and Applied Electromagnetics

This question paper has 12 pages (not including this cover page).

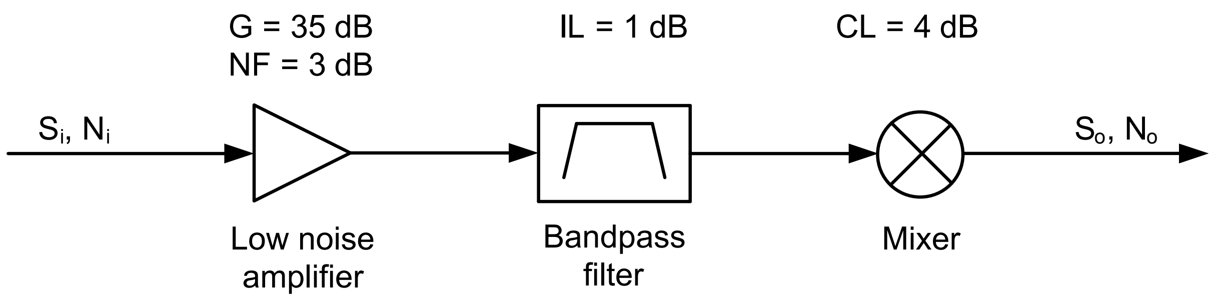
This question paper has 4 questions.

Answer a minimum of one question and at most three questions from this subject area.

Use a separate booklet (i.e., blue booklet) for the answers to questions in this subject area.

1. This question has 4 parts (a)-(d).

A wireless receiver is constituted by a low noise amplifier (LNA) with gain $G = 35$ dB and noise figure $NF = 3$ dB; and $P_{1dB} = 13$ dBm, a band pass filter having 1 dB insertion loss (IL), and a passive mixer having a conversion loss (CL) of 4 dB as shown in the lineup below. The receiver parameters are: noise reference temperature: $T_0 = 290$ K; signal bandwidth: $B = 1$ MHz; output third-order intercept point: $IP3 = 27$ dBm. Boltzmann constant $k = 1.38 \times 10^{-23}$J/K.



(a) Calculate the overall noise figure (NF), the gain and the equivalent noise temperature (T_e) of the receiver.

(b) Calculate the noise power at the output of the receiver.

(c) Calculate the dynamic range (DR) of the receiver.

(d) Find the carrier to the third-order intermodulation products ratio (C/IMD3) at the output the receiver when it is driven with a two-tone signal having a total power of 10 dBm.

2. This question has 12 parts (a)-(l).

At a sufficient distance from an antenna, the fields radiated by the antenna may be represented using a uniform plane wave. Assume that the fields are traveling in a source-free region of free space ($\epsilon = \epsilon_0$, $\mu = \mu_0$, $\sigma = 0$). The electric field is given by:

$$\mathbf{E}(x,t) = 100\cos\left(6\times 10^9 t - \beta x\right)\mathbf{a}_y \text{V/m}$$

Find the:

(a) frequency (f)

(b) wavelength (λ)

(c) phase constant (β)

(d) magnetic field ($\mathbf{H}(x,t)$)

The antenna system is being used for thru-wall inspection (i.e. waves travel through a wall and detect objects on the other side). The wall has $\epsilon_r = 3$, $\sigma = 0.1$ S/m and $\mu_r = 1$. Given that the wave is normally incident on the wall, find the:

(e) attenuation coefficient (α)

(f) phase constant (β)

(g) intrinsic impedance (η)

(h) reflection coefficient (Γ)

(i) transmission coefficient (T)

(j) velocity at which the wave travels in the wall

(k) For the incident field given above, find an expression for the electric ($\mathbf{E^r}(x,t)$) and magnetic ($\mathbf{H^r}(x,t)$) fields reflected from the wall

(l) For the incident field given above, find an expression for the electric ($\mathbf{E^t}(x,t)$) and magnetic ($\mathbf{H^t}(x,t)$) fields associated with the wave transmitted into the wall

3. This question has 2 parts (a)-(b).

A capacitor is made up of 2 concentric spheres with the inner sphere having radius a and a charge $+Q$ coulombs. The outer sphere has a radius b and a charge $-Q$ coulombs. NOTE that a, b and Q are constants.

The gap between the capacitors is divided into 3 equal spaces with permittivities of ϵ_1, $2\epsilon_1$ and $3\epsilon_1$.

(a) Using field analysis, derive an expression for the capacitance C of this capacitor.

(b) If the permittivity is $\epsilon_1 = \epsilon_0 \epsilon_r$ with $\epsilon_r = 2$ and $a = 5$ mm and $b = 9$ mm, what is the value of the capacitance of the concentric spheres.

4. Design a quarter wave transformer using rectangular waveguide technology. The transformer is to operate at 10 GHz and be positioned between a section of a waveguide with dimensions a=22.86 mm × b=10.16 mm (air filled), and a load with 800 Ω impedance (wave impedance).

   Design the transformer using a change in a broad dimension a. Ignore effects at the corners/junction between the transformer section and regular waveguide, in your design, but describe them qualitatively. Draw the structure, show all dimensions of the transformer.

Maxwell's Equations

integral form		point form
$\oint_C \vec{E} \cdot d\vec{L} = -\frac{d}{dt}\iint_S \vec{B} \cdot d\vec{S}$	**Faraday's Law**	$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$
$\oint_C \vec{H} \cdot d\vec{L} = I_c + \iint_S \frac{d\vec{D}}{dt} \cdot d\vec{S}$	**Ampere's Law**	$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$
$\oint_S \vec{D} \cdot d\vec{S} = Q = \iiint_V \rho_V dv$	**Gauss' Law for Electric Fields**	$\vec{\nabla} \cdot \vec{D} = \rho_v$
$\oint_S \vec{B} \cdot d\vec{S} = 0$	**Gauss' Law for Magnetic Fields**	$\vec{\nabla} \cdot \vec{B} = 0$

Electric Fields and Potential

Coulomb's Law

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 \left|\vec{R}_{12}\right|^2}\hat{a}_{12}$$

Electric Field

$$\vec{E}(\vec{r}) = \iiint_{V'} \frac{\rho_V(\vec{r}')}{4\pi\varepsilon_0} \frac{(\vec{r} - \vec{r}')}{\left|\vec{r} - \vec{r}'\right|^3} dv'$$

Work

$$W_E = -Q \int_{initial}^{final} \vec{E} \cdot d\vec{L}$$

Potential Difference

$$V_{AB} = -\int_A^B \vec{E} \cdot d\vec{L}$$

Potential field of distributed charges

$$V(\vec{r}) = \iiint_{V'} \frac{\rho_V(\vec{r}')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}'\right|} dV'$$

Electric Field from Potential

$$\vec{E} = -\vec{\nabla} V$$

Potential of a Dipole

$$V(\vec{r}) = \frac{\vec{p} \cdot (\vec{r} - \vec{r}')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}'\right|^3}$$

Fields from Sheet of Charge

$$\vec{E}(\vec{r}) = \frac{\rho_S}{2\varepsilon}\hat{a}_N$$

Fields from Line Charge

$$\vec{E}(\vec{r}) = \frac{\rho_L}{2\pi\varepsilon\rho}\hat{a}_\rho$$

Magnetic Fields, Forces, and Torque

Biot-Savart Law

$$\vec{H}(\vec{r}) = \begin{cases} \int_L \frac{I\, d\vec{L} \times (\vec{r} - \vec{r}')}{4\pi \left|\vec{r} - \vec{r}'\right|^3} \\ \iint_S \frac{\vec{K} \times (\vec{r} - \vec{r}')}{4\pi \left|\vec{r} - \vec{r}'\right|^3} dS' \\ \iiint_{V'} \frac{\vec{J} \times (\vec{r} - \vec{r}')}{4\pi \left|\vec{r} - \vec{r}'\right|^3} dv' \end{cases}$$

Fields from Line Current

$$\vec{H}(\vec{r}) = \frac{I}{2\pi\rho}\hat{a}_\phi$$

Fields from Sheet of Current Density

$$\vec{H}(\vec{r}) = \frac{1}{2}\vec{K} \times \hat{a}_N$$

Magnetic Force on Currents

$$\vec{F} = \oint_C I\, d\vec{L} \times \vec{B} \quad \text{(line current)}$$

$$\vec{F} = \iint_{S'} \vec{K} \times \vec{B} dS' \quad \text{(surface current density)}$$

$$\vec{F} = \iiint_{V'} \vec{J} \times \vec{B} dV' \quad \text{(volume current density)}$$

Lorentz Force Equation

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$$

Torque

$$\vec{T} = I\vec{S} \times \vec{B} = \vec{m} \times \vec{B}$$

Current and Conductors

Continuity of Current

integral form $\oint_S \vec{\boldsymbol{J}} \cdot \boldsymbol{d}\vec{\boldsymbol{S}} = -\frac{dQ_i}{dt}$

point form $\vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{J}} = -\frac{d\rho_V}{dt}$

Current Density

$$\vec{\boldsymbol{J}} = \sigma \vec{\boldsymbol{E}}$$

Material Relations and Boundary Conditions

$$\varepsilon_0 = 8.854 \times 10^{-12} \ \mathrm{F/m}$$

Dielectric Material Relations

$$\vec{\boldsymbol{D}} = \varepsilon_0 \vec{\boldsymbol{E}} + \vec{\boldsymbol{P}} = \varepsilon \vec{\boldsymbol{E}}$$

$$\varepsilon = (1 + \chi_e)\, \varepsilon_0 = \varepsilon_r \varepsilon_0$$

Electric Boundary Conditions

normal $\left(\vec{\boldsymbol{D}}_1 - \vec{\boldsymbol{D}}_2\right) \cdot \widehat{\boldsymbol{a}}_{21} = \rho_s$

tangential $\left(\vec{\boldsymbol{E}}_1 - \vec{\boldsymbol{E}}_2\right) \times \widehat{\boldsymbol{a}}_{21} = 0$

Angles at Interface

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

$$\left|\vec{\boldsymbol{D}}_2\right| = \left|\vec{\boldsymbol{D}}_1\right| \sqrt{\cos^2\theta_1 + \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 \sin^2\theta_1}$$

$$\left|\vec{\boldsymbol{E}}_2\right| = \left|\vec{\boldsymbol{E}}_1\right| \sqrt{\sin^2\theta_1 + \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 \cos^2\theta_1}$$

$$\mu_0 = 4\pi \times 10^{-7} \ \mathrm{H/m}$$

Magnetic Material Relations

$$\vec{\boldsymbol{B}} = \mu_0 \left(\vec{\boldsymbol{H}} + \vec{\boldsymbol{M}}\right) = \mu \vec{\boldsymbol{H}}$$

$$\mu = (1 + \chi_m)\, \mu_0 = \mu_r \mu_0$$

Magnetic Boundary Conditions

normal $\left(\vec{\boldsymbol{B}}_1 - \vec{\boldsymbol{B}}_2\right) \cdot \widehat{\boldsymbol{a}}_{21} = 0$

tangential $\widehat{\boldsymbol{a}}_{21} \times \left(\vec{\boldsymbol{H}}_1 - \vec{\boldsymbol{H}}_2\right) = \vec{\boldsymbol{K}}$

Polarization Charge Relations

bound surface charge density $\rho_{ps} = \vec{\boldsymbol{P}} \cdot \widehat{\boldsymbol{a}}_N$

bound volume charge density $\rho_{pv} = -\vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{P}}$

Circuit Parameters, Power and Energy

Resistance

$$R = \frac{V_{AB}}{I} = \frac{-\int_A^B \vec{\boldsymbol{E}} \cdot \boldsymbol{d}\vec{\boldsymbol{L}}}{\iint_S \vec{\boldsymbol{J}} \cdot \boldsymbol{d}\vec{\boldsymbol{S}}}$$

Power Loss (Joule/Ohmic Loss)

$$P_E = \iiint_{Vol} \left(\vec{\boldsymbol{J}} \cdot \vec{\boldsymbol{E}}\right) dv$$

Capacitance

$$C = \frac{Q}{V_{AB}} = \frac{\oint_S \vec{\boldsymbol{D}} \cdot \boldsymbol{d}\vec{\boldsymbol{S}}}{-\int_A^B \vec{\boldsymbol{E}} \cdot \boldsymbol{d}\vec{\boldsymbol{L}}} = \frac{2W_E}{V^2}$$

Energy Stored in Electric Field

$$W_E = \frac{1}{2}\iiint_{Vol} \left(\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{D}}\right) dv = \frac{1}{2}\iiint_{Vol} V_p \rho_v dv$$

Inductance

$$L = \frac{N\Psi}{I} = \frac{N\iint_S \vec{\boldsymbol{B}} \cdot \boldsymbol{d}\vec{\boldsymbol{S}}}{\oint_C \vec{\boldsymbol{H}} \cdot \boldsymbol{d}\vec{\boldsymbol{L}}} = \frac{2W_M}{I^2}$$

Energy Stored in Magnetic Field

$$W_M = \frac{1}{2}\iiint_{Vol} \left(\vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{H}}\right) dv$$

Cylindrical Coordinates

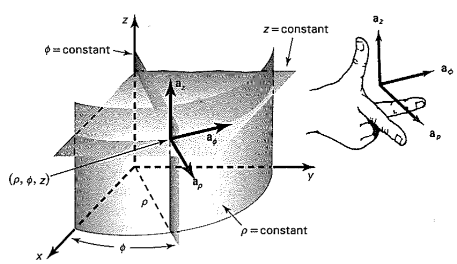
$$
\begin{aligned}
x &= \rho\cos\phi & \quad & \rho = \sqrt{x^2+y^2} \\
y &= \rho\sin\phi & \iff & \phi = \tan^{-1}\frac{y}{x} \\
z &= z & & z = z
\end{aligned}
$$

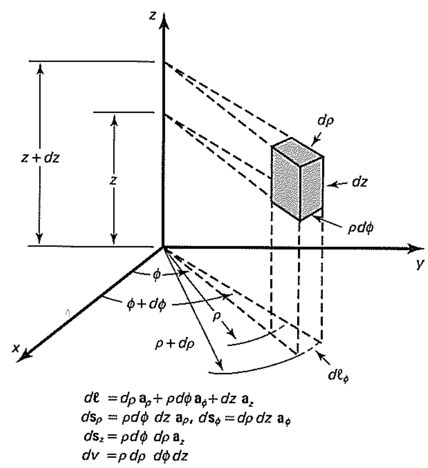
Field Component Transformations

$$
\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}
\qquad
\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}
$$

Differential Elements

Differential Elements	$d\vec{l} =$	$d\vec{S} =$ one of			$dv =$
	$d\rho\widehat{\boldsymbol{a}}_\rho + \rho d\phi\widehat{\boldsymbol{a}}_\phi + dz\widehat{\boldsymbol{a}}_z$	$(\rho d\phi dz)\,\widehat{\boldsymbol{a}}_\rho$	$(d\rho dz)\,\widehat{\boldsymbol{a}}_\phi$	$(\rho d\rho d\phi)\,\widehat{\boldsymbol{a}}_z$	$\rho d\rho d\phi dz$





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Spherical Coordinates

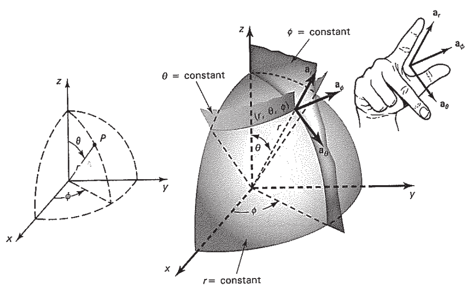
$$
\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}
\iff
\begin{aligned} r &= \sqrt{x^2+y^2+z^2} \\ \theta &= \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}} \\ \phi &= \tan^{-1}\frac{y}{x} \end{aligned}
$$

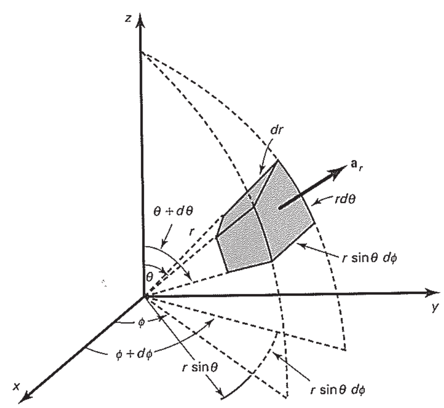
Field Component Transformations

$$
\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}
\qquad
\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}
$$

Differential Elements

Differential	$d\vec{l} =$	$d\vec{S} =$ one of	$dv =$
Elements	$dr\widehat{a}_r + rd\theta\widehat{a}_\theta + r\sin\theta d\phi\widehat{a}_\phi$	$\left(r^2\sin\theta d\theta d\phi\right)\widehat{a}_r$ $\quad (r\sin\theta dr d\phi)\,\widehat{a}_\theta$ $\quad (r dr d\theta)\,\widehat{a}_\phi$	$r^2\sin\theta dr d\theta d\phi$





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Vector Operators

		Cartesian	Cylindrical
Gradient	$\vec{\boldsymbol{\nabla}} F =$	$\frac{\partial F}{\partial x}\widehat{\boldsymbol{a}}_x + \frac{\partial F}{\partial y}\widehat{\boldsymbol{a}}_y + \frac{\partial F}{\partial z}\widehat{\boldsymbol{a}}_z$	$\frac{\partial F}{\partial \rho}\widehat{\boldsymbol{a}}_\rho + \frac{1}{\rho}\frac{\partial F}{\partial \phi}\widehat{\boldsymbol{a}}_\phi + \frac{\partial F}{\partial z}\widehat{\boldsymbol{a}}_z$
Divergence	$\vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{F}} =$	$\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial\left(\rho F_\rho\right)}{\partial \rho} + \frac{1}{\rho}\frac{\partial\left(F_\phi\right)}{\partial \phi} + \frac{\partial F_z}{\partial z}$
Curl	$\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{F}} =$	$\begin{vmatrix} \widehat{\boldsymbol{a}}_x & \widehat{\boldsymbol{a}}_y & \widehat{\boldsymbol{a}}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$	$\frac{1}{\rho}\begin{vmatrix} \widehat{\boldsymbol{a}}_\rho & \rho\widehat{\boldsymbol{a}}_\phi & \widehat{\boldsymbol{a}}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_\rho & \rho F_\phi & F_z \end{vmatrix}$
Laplacian	$\nabla^2 F =$	$\frac{\partial^2 F_x}{\partial x^2} + \frac{\partial^2 F_y}{\partial y^2} + \frac{\partial^2 F_z}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial F_\rho}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 F_\phi}{\partial \phi^2} + \frac{\partial^2 F_z}{\partial z^2}$

		Spherical
Gradient	$\vec{\boldsymbol{\nabla}} F =$	$\frac{\partial F}{\partial r}\widehat{\boldsymbol{a}}_r + \frac{1}{r}\frac{\partial F}{\partial \theta}\widehat{\boldsymbol{a}}_\theta + \frac{1}{r\sin\theta}\frac{\partial F}{\partial \phi}\widehat{\boldsymbol{a}}_\phi$
Divergence	$\vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{F}} =$	$\frac{1}{r^2}\frac{\partial\left(r^2 F_r\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\left(\sin\theta F_\theta\right)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$
Curl	$\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{F}} =$	$\frac{1}{r^2\sin\theta}\begin{vmatrix} \widehat{\boldsymbol{a}}_r & r\widehat{\boldsymbol{a}}_\theta & r\sin\theta\widehat{\boldsymbol{a}}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ F_r & rF_\theta & r\sin\theta F_\phi \end{vmatrix}$
Laplacian	$\nabla^2 F =$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial F_r}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial F_\theta}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 F_\phi}{\partial \phi^2}$

$\mu_o = 4\pi \times 10^{-7}$ H/m
$1/\mu_o = 8 \times 10^5$ m/H
$\eta_o = 120\pi\ \Omega$
$\varepsilon o = 8.85\times10^{-12}$ F/m

$\mathbf{a}_r \cdot \mathbf{a}_x = \sin\theta\cos\phi$	$\mathbf{a}_\theta \cdot \mathbf{a}_x = \cos\theta\cos\phi$	$\mathbf{a}_\varphi \cdot \mathbf{a}_x = -\sin\phi$
$\mathbf{a}_r \cdot \mathbf{a}_y = \sin\theta\sin\phi$	$\mathbf{a}_\theta \cdot \mathbf{a}_y = \cos\theta\sin\phi$	$\mathbf{a}_\varphi \cdot \mathbf{a}_y = \cos\phi$
$\mathbf{a}_r \cdot \mathbf{a}_z = \cos\theta$	$\mathbf{a}_\theta \cdot \mathbf{a}_z = -\sin\theta$	$\mathbf{a}_\varphi \cdot \mathbf{a}_z = 0$

$\mathbf{a}_x \cdot \mathbf{a}_\rho = \cos\phi$	$\mathbf{a}_x \cdot \mathbf{a}\phi = -\sin\phi$	$\mathbf{a}_x \cdot \mathbf{a}_z = 0$
$\mathbf{a}_y \cdot \mathbf{a}_\rho = \sin\phi$	$\mathbf{a}_y \cdot \mathbf{a}\phi = \cos\phi$	$\mathbf{a}_y \cdot \mathbf{a}_z = 0$
$\mathbf{a}_z \cdot \mathbf{a}_\rho = 0$	$\mathbf{a}_z \cdot \mathbf{a}\phi = 0$	$\mathbf{a}_z \cdot \mathbf{a}_z = 1$

$x = \rho \cos\phi$

$y = \rho \sin\phi$

Cartesian	Cylindrical	Spherical
$dx\,\boldsymbol{ax} + dy\,\boldsymbol{ay} + dz\,\boldsymbol{az}$	$d\rho\,\boldsymbol{a}\rho + \rho d\phi\,\boldsymbol{a}\phi + dz\,\boldsymbol{az}$	$dr\,\boldsymbol{ar} + r\,d\theta\,\boldsymbol{a}\theta + r\sin\theta\,d\phi\,\boldsymbol{a}\phi$
$(dydz)\boldsymbol{ax}$	$(\rho\,d\phi\,dz)\boldsymbol{a}\rho$	$(r^2\sin\theta\,d\theta\,d\phi)\boldsymbol{ar}$
$(dxdz)\mathbf{a}y$	$(d\rho\,dz)\boldsymbol{a}\phi$	$(r\sin\theta\,dr\,d\phi)\boldsymbol{a}\theta$
$(dxdy)\boldsymbol{az}$	$(\rho\,d\rho\,d\phi)\boldsymbol{az}$	$(r\,dr\,d\theta)\boldsymbol{a}\phi$
$dx\,dy\,dz$	$\rho\,d\rho\,d\phi\,dz$	$r^2\sin\theta\,dr\,d\theta\,d\phi$

$r = \sqrt{x^2 + y^2 + z^2}$ $\quad x = r\sin\theta\cos\phi$

$\theta = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}}$ $\quad y = r\sin\theta\sin\phi$

$\phi = \tan^{-1}\frac{y}{x}$ $\quad z = r\cos\theta$

$$\nabla V = \frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\bar{a}_\phi + \frac{\partial V}{\partial z}\bar{a}_z$$

$$\nabla V = \frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\bar{a}_\phi$$

$$\nabla\cdot\vec{E} = \left[\frac{\partial(E_x)}{\partial x} + \frac{\partial(E_y)}{\partial y} + \frac{\partial(E_z)}{\partial z}\right]$$

$$\nabla\cdot\vec{E} = \left[\frac{1}{\rho}\frac{\partial(\rho E_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial(E_\phi)}{\partial \phi} + \frac{\partial(E_z)}{\partial z}\right]$$

$$\nabla\cdot\vec{E} = \left[\frac{1}{r^2}\frac{\partial(r^2 E_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta E_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial(E_\phi)}{\partial \phi}\right]$$

$$\nabla^2 V = \left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right]$$

$$\nabla^2 V = \left[\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho\frac{\partial V}{\partial \rho}) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + + \frac{\partial^2 V}{\partial z^2}\right]$$

$$\nabla^2 V = \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial V}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial V}{\partial \theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}\right]$$

$$\nabla\times\overline{\mathbf{A}} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \mathbf{A}_x & \mathbf{A}_y & \mathbf{A}_z \end{vmatrix}$$

$$\nabla\times\overline{\mathbf{A}} = \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ \mathbf{A}_\rho & \rho\mathbf{A}_\phi & \mathbf{A}_z \end{vmatrix}$$

$$\nabla\times\overline{\mathbf{A}} = \frac{1}{r^2\sin\theta}\begin{vmatrix} a_r & r a_\theta & r\sin\theta a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \mathbf{A}_r & r\mathbf{A}_\theta & r\sin\theta\mathbf{A}_\phi \end{vmatrix}$$

Magnetostatics	materials
$\vec{H} = \int_L \frac{Id\vec{l} \times \vec{a}_R}{4\pi R^2}$ $W_M = \frac{1}{2}\int \vec{H} \bullet \vec{B} dv$ **T**=**m** x **B** $L = \frac{N\Psi}{I}$ $M_{12} = \frac{N_1\Psi_{12}}{I_2}$ $\vec{F} = \oint I d\vec{l} x \vec{B}$	$\vec{M} = \chi_m \vec{H}$ $\nabla \times \vec{M} = \vec{J}_M$ $\mu_r=1+\chi_m$
Lorentz force **F**=q(**E**+**v**x**B**)	Continuity $\nabla \bullet \vec{J} = -\frac{\partial \rho_v}{\partial t}$
Electrostatics	Materials
$\vec{E}(\vec{r}) = \int \frac{\rho_v dv}{4\pi\varepsilon_0\varepsilon_r R^2}\vec{a}_R$ $V(\vec{r}) = \int \frac{\rho_v(\vec{r}')dv'}{4\pi\varepsilon_r\varepsilon_0 \lvert \vec{r} - \vec{r}' \rvert} + C$ $V = -\int \vec{E} \bullet d\vec{l} + C$ $\vec{E} = -\nabla V$ $W_E = \frac{1}{2}\int \vec{E} \bullet \vec{D} dv$ $\nabla^2 V = -\frac{\rho_v}{\varepsilon}$ $\nabla^2 V = 0$ C=Q/V	$R = l/(\sigma S)$ $\vec{P} = \chi_e \varepsilon_o \vec{E}$ $\nabla \bullet \vec{P} = -\rho_{pv}$ $\vec{P} \bullet \vec{a}_n = \rho_{ps}$ $\varepsilon_r=1+\chi_e$

Boundary conditions		
$\mathbf{D}_{1n}-\mathbf{D}_{2n}=\rho s$ $\mathbf{E}_{t1}=\mathbf{E}_{t2}$ $\mathbf{B}_{1n}-\mathbf{B}_{2n}=0$ $\mathbf{a}_{21} \times (\mathbf{H}_1-\mathbf{H}_2)=\mathbf{J}_s$	$\mathbf{a}_{21} \cdot (\mathbf{D}_1-\mathbf{D}_2)=\rho s$ $\mathbf{a}_{21} \times (\mathbf{E}_1-\mathbf{E}_2)=0$ $\mathbf{a}_{21} \cdot (\mathbf{B}_1-\mathbf{B}_2)=0$ $\mathbf{a}_{21} \times (\mathbf{H}_1-\mathbf{H}_2)=\mathbf{J}_s$	$\mathbf{J}_s$ is surface current (also denoted as **K**)

Maxwell's equations	
$\oint_c \vec{E} \bullet d\vec{l} = -\frac{\partial}{\partial t}\int_s \vec{B} \bullet d\vec{s}$ $\oint_s \vec{B} \bullet d\vec{s} = 0$ $\oint_s \varepsilon_r \varepsilon_0 \vec{E} \cdot d\vec{s} = \int_v \rho_v dv$ $\oint_c \vec{H} \bullet d\vec{l} = \int_s \vec{J} \bullet d\vec{s} + \frac{\partial}{\partial t}\int_s \vec{D} \bullet d\vec{s}$ $\nabla \bullet \vec{B} = 0$ $\nabla \bullet \vec{D} = \rho_v$ $\nabla \times \vec{H} = \vec{J} + \frac{\partial}{\partial t}\vec{D}$ $\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$ $V_{emf} = -\int_s \frac{\partial \vec{B}}{\partial t} \bullet d\vec{s} + \oint_L (\vec{v} x \vec{B}) \bullet d\vec{l}$	Note: for static fields, time derivatives are zero. In phasor form, time derivatives become $j\omega$ terms.

Time-varying fields: UPW	
$\nabla^2 \vec{E}_s - \gamma^2 \vec{E}_s = 0$ $\nabla^2 \vec{H}_s - \gamma^2 \vec{H}_s = 0$ $\gamma = \alpha + j\beta$	Vector wave equations for time-harmonic fields in lossy medium. With lossless medium or free space, $\alpha=0$.
$\lambda = \frac{2\pi}{\beta}$ $\beta = \omega\sqrt{\mu_r\mu_o\varepsilon_r\varepsilon_o}$ $T = 1/f$ $\lvert E \rvert / \lvert H \rvert = \eta = \sqrt{\frac{\mu_r\mu_o}{\varepsilon_r\varepsilon_o}}$ $v_p = \frac{\omega}{\beta}$	Uniform plane wave in lossless medium. For free space, $\mu r=1$ and $\varepsilon r=1$.
$\vec{E}(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\vec{a}_x$ $\vec{H}(z,t) = \frac{E_o}{\lvert\eta\rvert} e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\vec{a}_y$	One example of E and H fields in lossy medium.
$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$ $\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$ $\lvert\eta\rvert = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$ $\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$ $\delta=1/\alpha$ $\lambda = \frac{2\pi}{\beta}$	Parameters describing UPW in lossy medium.
$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$ $\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^o$	good conductor: $(\sigma/\omega\varepsilon >> 1)$
$\vec{P}_{avg}(z) = \frac{1}{2}\mathrm{Re}(\vec{E}_s(z) \times \vec{H}_s^*(z))$ **P**(z,t)=**E**(z,t) x **H**(z,t)	Poynting vector
$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ $T = \frac{2\eta_2}{\eta_2 + \eta_1}$	Transmission and reflection coefficients: normal incidence
$\Gamma_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$ $T_\parallel = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$ $\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$ $T_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$	Transmission and reflection coefficients: oblique incidence $\theta_i = \theta_r$ $\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}$

Waves and T/R – continued

$$s = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\sin\theta_B = \sqrt{\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}}$$

$$\tan\theta_B = \sqrt{\varepsilon_2/\varepsilon_1}$$

$$\sin\theta_c = \sqrt{\varepsilon_2/\varepsilon_1}$$

$$R_{ac} = L/(\sigma\delta w)$$

Transmission lines:

$$V_s(z) = V^+e^{-\gamma z} + V^-e^{\gamma z}$$

$$I_s(z) = \frac{1}{Z_o}\left[V^+e^{-\gamma z} - V^-e^{\gamma z}\right]$$

$$Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\gamma = \sqrt{(G+j\omega C)(R+j\omega L)} = \alpha + j\beta$$

$$P_{ave} = \frac{|V_o^+|^2}{2Z_o}e^{-2\alpha z}\cos\theta$$

Distortionless transmission lines:

R/L=G/C

$$\alpha = \sqrt{RG}$$

$$\beta = \omega\sqrt{LC}$$

$$R_o = \sqrt{\frac{R}{G}}$$

Lossless line:

$$Z_{in} = Z_o\frac{Z_L + jZ_o\tan(\beta l)}{Z_o + jZ_L\tan(\beta l)}$$

$$Z_{in\_max} = Z_o\frac{1+|\Gamma|}{1-|\Gamma|} = Z_o \cdot SWR$$

$$\Gamma_l = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_{in\_min} = \frac{Z_o}{SWR}$$

$$\Gamma(l) = \Gamma(0)e^{-j2\beta l}$$

$$SWR = \left|\frac{V_{max}}{V_{min}}\right| = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Waveguides:

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_p = u_p = \frac{u'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_g = u_g = u'\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$v_p v_g = u'^2$$

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\alpha_c|_{TE10} = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left(0.5+\frac{b}{a}\left(\frac{f_c}{f}\right)^2\right)$$

$$\alpha_c|_{TE} = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left(\left(1+\frac{b}{a}\right)\left(\frac{f_c}{f}\right)^2 + \frac{\frac{b}{a}\left(\frac{b}{a}m^2+n^2\right)}{\frac{b^2}{a^2}m^2+n^2}\left(1-\left(\frac{f_c}{f}\right)^2\right)\right)$$

$$\alpha_c|_{TM} = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left(\frac{\left(\frac{b^3}{a^3}m^2+n^2\right)}{\frac{b^2}{a^2}m^2+n^2}\right)$$